Pre-service Secondary Teachers' Level in Proving Tasks

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Abstract

This paper documented pre-service secondary teachers' levels of proof writing in the Philippines. Construction of proof is an important skill for those who have interest in mathematics. However, literature revealed that proof construction is widely acknowledged to be one of the most challenging aspects in the mathematics curriculum. A sample of 28 pre-service secondary teachers participated in the study. Data were collected by means of written tests and focus group discussions. Most of them applied empirical approaches, where an assertion is valid from a small number of cases only, in their written proofs and left some items unanswered. The results of the study indicate that there is a necessity to conduct further investigations into the underlying reasons to strengthen the pre-service teachers' ability to provide quality education to students, as pre-service teachers play a critical role in shaping the future of education and ensuring that every student has equal access to a high-quality education.

Keywords: pre-service teachers, proof level, construction of proof, mathematics education, quality education, SDG 4

1. INTRODUCTION

Aristotle once said that "man is a rational animal". Hence, human nature is set apart from other terrestrial creatures by their rationality. According to Cooper (n.d.), written proofs are ways to reflect understanding, which through this; everyone can communicate mathematical ideas with other people. He also pointed out that engaging and "doing" mathematics are all about finding proofs and applying these in real life. Particularly in mathematics as stated in the K to 12 Mathematics Curriculum Guide by the Department of Education (2013), proving is a skill of showing the existence, truthfulness or falsity of a mathematical statement by using logical reasoning and arguments. Thus, it offers powerful ways of understanding the true nature of a phenomenon in a very clear way. In order to address the global competitiveness of every Filipinos and the integration of ASEAN 2015, the K to 12 Basic Education Curriculum was developed and made into law through Republic Act No. 10533, entitled "Enhanced Basic Education Act of 2013". RA 10533 provides that the State shall create a functional basic education system that will develop productive and responsible citizens equipped with the essential competencies, skills and values for both life – long learning and employment. Critical Thinking and Problem-Solving are the twin goals to be achieved in learning Mathematics in the K to 12 Basic Education Curriculum (BEC). Mathematical Problem-Solving as one of the twin goals of the K to 12 BEC was defined as finding a way around a difficulty or obstacle, and finding a solution to a problem that is unknown as cited from Polya (1945, 1962). On the other hand, critical thinking, as cited from Scriven and Paul (1987), is an intellectually disciplined process of actively and skillfully conceptualizing, applying, analyzing, synthesizing, and/or evaluating information gathered from, or generated by, observation, experience, reflection, reasoning, or communication, as a guide to belief and action. These twin goals are incorporated in the five content areas of the curriculum, namely, Numbers and Number Sense, Measurement, Geometry, Patterns and Algebra, and Probability and Statistics. One of the specific skills to be developed in the mathematics curriculum is proving. According to Harel and Sowder (1998), conception of proof can be classified into three proof schemes:

1. **External Conviction Proof Schemes**, where arguments are based on sources external to the *symbols' meaning (e.g. the word of an authority);*
2. **Empirical Proof Scheme**, where it is characterized by using examples or specific cases as the basis for an argument; and
3. **Analytical Proof Scheme**, it is characterized when the argument is based on the use of deduction. Further, it can be either transformational or axiomatic. A transformational proof scheme involves goal-oriented operations on objects where an individual operates with a deductive process in which he or she considers generality aspects, applies goal-oriented and anticipatory mental operations, and transforms images. But an axiomatic proof scheme goes beyond a transformational one in that the student also recognizes that mathematical systems rest on (possibly arbitrary) statements that are accepted without proof.

The classification above was further developed by Zeybek (2016) in her paper into the following six (6) proofs levels:

1. **External Reasoning**. Responses appeal to external authority. The subcategories are Authoritarian proof, Ritual proof, and non-referential symbolic proof which depends on an authority, the appearance of the argument, and on some symbolic manipulation, respectively.
2. **Naive Reasoning.** Responses may appeal to the use of some deduction, something that provers remember hearing often incorrect, but ends with incorrect conclusion or correct conclusion with incorrect reasons.
3. **Empirical Reasoning.** Responses appeal to empirical demonstrations, or rudimentary

transformational frame. The subcategories are Naive Empiricism, which is an assertion is valid from a small number of cases, and Crucial Empiricism, which is an assertion, is valid from strategically chosen cases of examples.

4. **Reasoning Based on Particular Instances.** Responses appeal to using properties inferred from a particular instance
5. **Incomplete Deductive Reasoning.** Responses appeal to a chain of inferences that is almost entirely deductive or complete. However, few inferences used in the justification may have some non-deductive reasoning, and may require further justifications or have minor errors.
6. **Deductive Reasoning.** Responses appeal to rigorous and logical reasoning. The subcategories are Transformational proof scheme, which involves goal-oriented operations on objects, and Axiomatic proof scheme, which involves statements that do not require justification.

1.1 Difficulties with Proofs for both Teachers and Students

Piaget and van Hiele provide understanding on how individuals can develop conceptions of proof. Piaget's theory focuses in what manner does proof happens without taking into consideration the curricula. On the other hand, van Hiele examines improvement in proof skills with curricula. The theories of Piaget and van Hiele are very important for us to understand an individual on how they construct and come up with their proofs (Crowley, 1987; Kendler & Kendler, 2008).

There are three stages that are essential in concept formation such as perceptual, conceptual and abstraction. The first stage is perceptual; this is when a person doesn't know the concept that is being asked in the question. Thus, in terms of proof and reasoning this is the stage where a person didn't understand the concept that is being asked. Then, conceptual is the second stage where a person knows to some degree about the idea but failed to apply it. Abstraction is the last or the third stage in Piaget's theory of reason and proof. In this stage, there is already an understanding of an individual on how he will execute with the concept in consideration (Kendler & Kendler, 2008).

During 1980's, two Dutch middle school teachers and researchers Dina van Hiele-Geldhof and Pierre van Hiele from Netherlands created the van Hiele model of the development of geometric thought. They researched about why an individual struggles in coming up with a proof. The van Hieles outlined five levels are as follows (Breyfogle & Lynch, 2010):

1. **Visualization.** A person recognizes figures by appearance alone, often by comparing them to a

known prototype.

2. **Analysis.** A person sees figures as collections of properties.
3. **Abstraction.** A person perceives relationships between properties and between figures.
4. **Deduction**. An individual can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions.
5. **Rigor.** A person at this level understands the formal aspects of deduction, such as establishing and comparing mathematical systems.

Here, an individual must progress through these levels chronologically (Connolly, 2010). Mutually, students and teachers play a significant role in the teaching and learning process especially in proving. Van Hieles ideas were developed to help teachers in structuring varied activities and organizing classroom instruction to develop student's competence in Mathematics thinking (Breyfogle & Lynch, 2010). Competence in proving in mathematics is one of the vital attributes that students have to acquire as stated in the K to 12 Mathematics Curriculum. However, there are various researchers who pointed out student's difficulty in proving (Derek, 2011; Heinze & Reiss, 2003; Liu & Manouchehri, 2013; Stavrou, 2014; Varghese, 2009). It was also observed that many students lack the knowledge of how to start a proof (Zeybek, 2016). The van Hieles proposed that individuals' progress chronologically from one level to the next by working through strategies, organization of instruction, content, materials which are appropriate in terms of language and task for their level of understanding. This is the reason why they come up with van Hieles five instructional phases as follows (Connolly, 2010):

Phase 1: Inquiry/Information. At this initial stage, the teacher and students engage in conversation and activity about the objects of study for this level.

Phase 2: Directed Orientation. The students explore the topic of study through materials that the teacher has carefully sequenced.

Phase 3: Explication Building. On their previous experiences, students express and exchange their emerging views about the structures that have been observed.

Phase 4: Free Orientation. The student encounters more complex tasks-tasks with many steps, tasks that can be completed in several ways, and open-ended tasks.

Phase 5: Integration. The students review and summarize what they have learned with the goal of forming an overview of the new network of objects and relations.

Likando and Ngoepe (2014) confer that in Mathematics, knowledge of axioms is the right set of accepted statements. Furthermore, student's proofs were either empirical in nature or lacked of axioms and definitions which are the very reasons why the conceptions of proof were underdeveloped. It is also evident in the study of Heinze & Reiss (2003) that students also have difficulties to bridge the gap between empirical and formal argumentation. Additionally, Derek (2011) discerned that students didn't realize the importance of the definitions and axioms which is also a cause of difficulty in proving. Meanwhile, it was found out that not only students but teachers as well have difficulty when proof becomes part in mathematics classroom activities (Knuth, 2002; Marriotti, 2006; Menezes, Viseu, Martins, & Gomes, 2015; Skott, Larsen, & Ǿstergaard, 2016). If one wishes to improve the abilities of students, the position of this study is that the best intervention to address this difficulty is to develop the courses in the tertiary level for the pre-service teachers.

Pre-service teachers are expected to be good in proving because they are the future teachers of the generation. It is but right and just to conduct researches and training for them to improve and cultivate teaching pedagogies and content knowledge throughout teacher education program because this is really a big help for the pre-service teachers. Also, lack of teaching pedagogies and content knowledge inhibited their ability to evaluate the validity of the conjectures and in presenting arguments (Zeybek, 2016). Braza & Supapo (2014) identified that teacher's difficulty of the basic concepts and skills are one of the challenges encountered in the implementation of the K to 12 Mathematics Curriculum. Despite of the importance of proof in mathematics, there are only few researches that has been conducted to investigate pre-service teachers' conceptions of proof, who are just few steps away from being a full pledged teacher (Ko, 2010; Zeybek, 2016).

This study aims to contribute in strengthening the quality of mathematics education in the Philippines specifically for pre-service secondary teachers, in the conceptions of proof, who will be the future mathematics teachers in the K to 12 Basic Education Curriculum in the country. Likewise, this study is also pursuant to DepEd Order No. 39, s. 2016 entitled "Adoption of the Basic Education Research Agenda" on how can pre-service teacher education be improved to develop teachers who can effectively deliver the K to 12 curriculums. Particularly, this study investigates the following questions:

1.1.1 What are pre–*service secondary teachers' levels in Proving tasks?*

1.1.2 What are the common errors that pre–service secon*dary teachers' in writing* mathematical proof?

2. METHODOLOGY

A descriptive research design was used in the study to determine the levels and competence of pre–

service secondary teachers about Mathematical Proofs. It is an appropriate design since this study deals with recording and tabulating data to come up with the results in identifying the levels of pre-service teachers in proving task. Further, this study uses a qualitative approach mainly on respondents' responses gathered in the Focus Group Discussion, which will be described below.

2.1 Data Collection and Analysis

Respondents consist of the 28 pre-service secondary mathematics teachers who are currently enrolled at one of the universities in the Philippines. The respondents in this study, in a voluntary participation, were fully aware of the purpose of the study and how the findings will be used. The information will not be made available to, or accessed by anybody, no part of this study by which it ensures such identifying information is excluded. Further, the researcher used pseudonyms for the names of the respondents. Lastly, respect for the dignity of respondents was given importance. The collection of data was divided into two phases: (1) Written test and (2) Focus Group Discussion (FGD).

2.1.1 Written Test

Each pre–service teacher will answer a 5 – item proving task for an hour. This is to identify the proof level of pre–service teachers' using Zeybek (2016) taxonomy of proof levels as well as to identify the common errors incurred, if there is any.

2.1.2 Focus Group Discussion

Respondents were divided into four groups having at least 10 members in each group in a FGD for about an hour. Open-ended questions will be given to the participants during the FGD which will be carried out at the same day. The recorded responses will be transcribed and analyzed.

2.1.3 Research Reflexivity

The Researcher's reflection of this research is to focus on the proof level of pre-service teachers to help in designing mathematics courses in teacher education institutions. Hence, the derived inputs or knowledge gained from this study will help future researchers enhance educational approaches, processes, and systems.

3. FINDINGS AND DISCUSSIONS

In this section, the researcher analyzed the mathematical proof constructed, notion of proof, and the common errors made by the pre-service secondary teachers in the written test and interview. It was found out that most of the items were left unanswered by the respondents. Those items with answers were classified using taxonomy of proof levels by Zeybek (2016) to identify the proof level of the respondents as presented below.

3.1 Written Test

Table 1.

Pre-ser*vice Secondary Teachers' Proof Levels*

Categories		Number of Respondents at this Category — Item Number					Total
		1	**2**	**3**	**4**	**5**	
External Reasoning Responses	Authoritarian Proof	2	1	0	0	0	**3**
	Ritual Proof	0	0	0	0	0	**0**
	Non – Referential Symbolic Proof	0	0	0	0	0	**0**
Naïve Reasoning		3	4	1	3	8	**18**
Empirical Reasoning	Naïve Empiricism	0	0	0	0	0	**0**
	CrucialEmpiricism	5	8	3	2	0	**19**
Reasoning Based on Particular Instances		0	0	0	0	0	**0**
Incomplete Deductive Reasoning		5	0	0	0	3	**8**
Deductive Reasoning	Transformational Proof Scheme	1	0	0	0	0	**1**
	Axiomatic Proof Scheme	8	0	0	1	0	**9**

As shown in the table above, most of the proofs made by pre-service teachers were empirical in nature, i. e., giving examples as proof. Also, they made proofs with correct conclusion but with incorrect reasoning which can be considered as naïve reasoning. But only ten (10) proofs can be considered as using deductive reasoning which appeals to rigorous and logical reasoning.

3.2 Pre-service Secondary Teachers' Construction of Proof

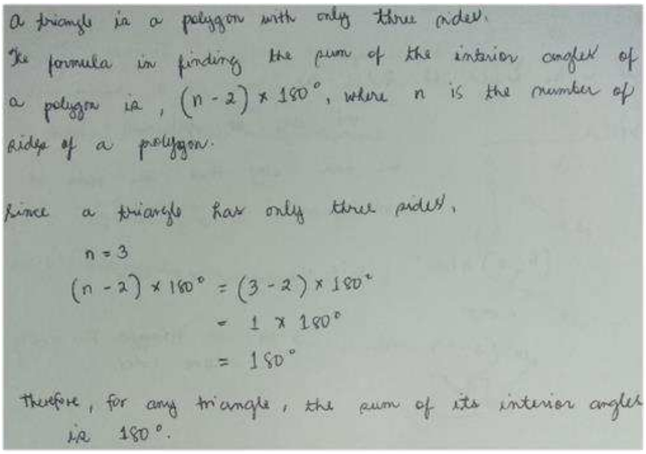
In this part, construction of pre-service teachers' construction of proof will be illustrated. All names used were only pseudonyms.

3.2.1 External Reasoning Responses

Mark argued in his proof that the given mathematical statement is true using a mathematical formula.

See Mark's proof below on showing that the sum of the interior angles of a triangle is 180°:

Figure 1.
Mark's Proof



a triangle is a polygon with only three sides.
The formula in finding the sum of the interior angles of a polygon is, $(n-2) \times 180°$, where n is the number of sides of a polygon.

Since a triangle has only three sides,

$n = 3$

$(n-2) \times 180° = (3-2) \times 180°$
$= 1 \times 180°$
$= 180°$

Therefore, for any triangle, the sum of its interior angles is 180°.

Mark's argument is based on a theorem that the sum of the interior angles of polygons is given by the formula (n-2)180° without using deductive and logical reasoning but merely using the authoritative aspect of a reference.

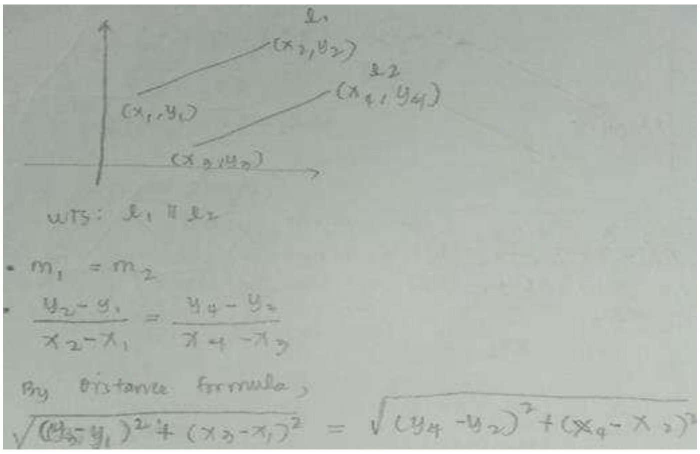
3.2.2 Naive Reasoning

The proofs made by pre-service teachers fall under this category. It was observed that they made a proof using deductive reasoning and come up with a correct conclusion but with incorrect reasons as illustrated below by proof of Anna on:

If ℓ_1 and ℓ_2 are two distinct nonvertical lines having slopes m_1 and m_2, respectively, then ℓ_1 and ℓ_2 are parallel if $m_1 = m_2$.

Figure 2.

Anna's Proof



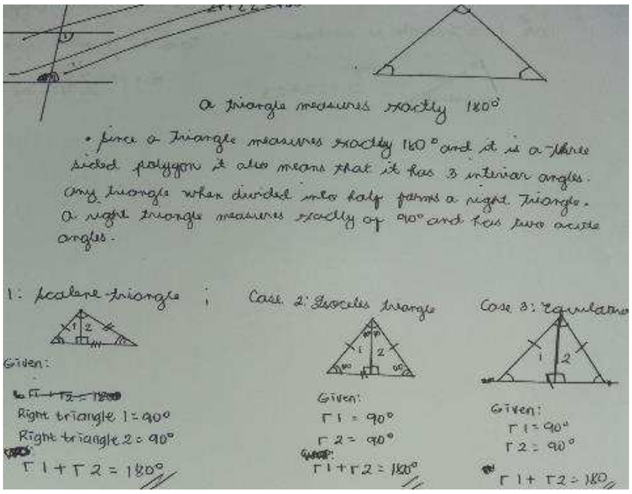
Anna's proof lacked steps in showing that two lines are parallel if their slopes are equal in the content of Euclidean Geometry. She just equated the two slopes to be equal and using the distance formula without consideration on what point she will be using in her proof.

3.2.3. Empirical Reasoning

Most of the proofs, i.e., 19 proofs, made by the respondents purposefully chose examples that will satisfy the given mathematical statement which classifies it as crucial empiricism. They argued that since the examples they presented satisfy the given conditions in the mathematical statement, hence they concluded that the claim is true. A proof made by a Carlo is shown below on showing that the sum of the of the interior angles of a triangle is 180°.

Figure 3.

Carlo's Proof



Carlo proved the mathematical statement using three (3) cases of different kinds of triangle whose interior angles sum up to 180°. But showing an example is not sufficient to claim the truthfulness of a mathematical statement. It must be proven for all cases to claim generality. Thus, using example to prove the mathematical statement is a common error among the pre-service teachers in the 5-item written test having a total of 19 responses that fall under this scheme.

3.2.4 Incomplete Deductive Reasoning

Respondents attempt to prove the given mathematical statement using a chain of deductive reasoning but made some minor errors and needs further justification in their claim. An illustration is shown below:

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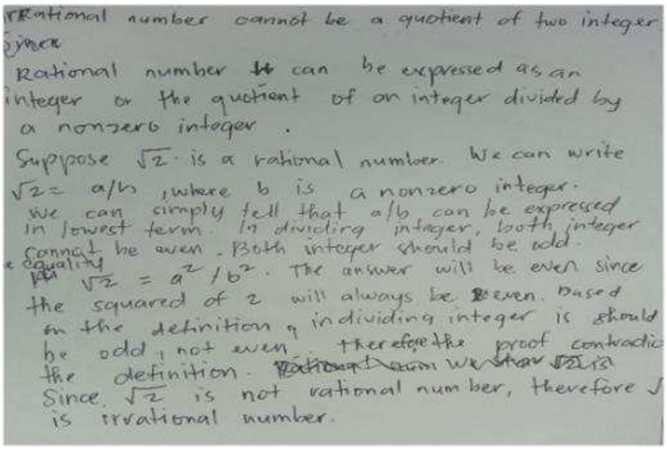
Figure 4.

Paulo's Proof



Irrational number cannot be a quotient of two integer
since
Rational number can be expressed as an
integer or the quotient of an integer divided by
a nonzero integer .
Suppose $\sqrt{2}$ is a rational number. We can write
$\sqrt{2} = a/b$, where b is a nonzero integer.
We can simply tell that a/b can be expressed
in lowest term. In dividing integer, both integer
cannot be even. Both integer should be odd.
Equality
$\sqrt{2} = a^2/b^2$. The answer will be even since
the squared of 2 will always be even. Based
on the definition, in dividing integer is should
be odd, not even. therefore the proof contradic
the definition.
Since $\sqrt{2}$ is not rational number, therefore $\sqrt{}$
is irrational number.

Paulo tried to prove in a logical way that $\sqrt{2}$ is an irrational number. He claimed that both a and b were both odd instead saying that they are relatively prime. He needs also to find the value of both a and b to come up with a contradiction.

3.2.5 Deductive Reasoning

Only ten (10) proofs were classified under this level where responses were made using logical chain of reasoning. Among the proofs it was classified further as transformational proof and axiomatic proof schemes where only one (1) proof fall under the transformational proof scheme of Leah as shown below.

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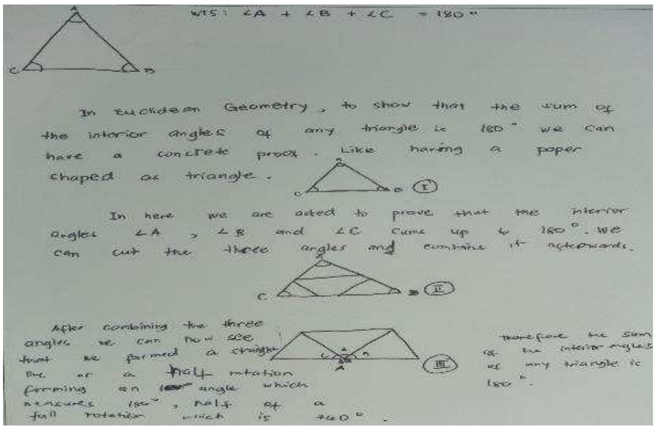
Figure 5.

Leah's Proof

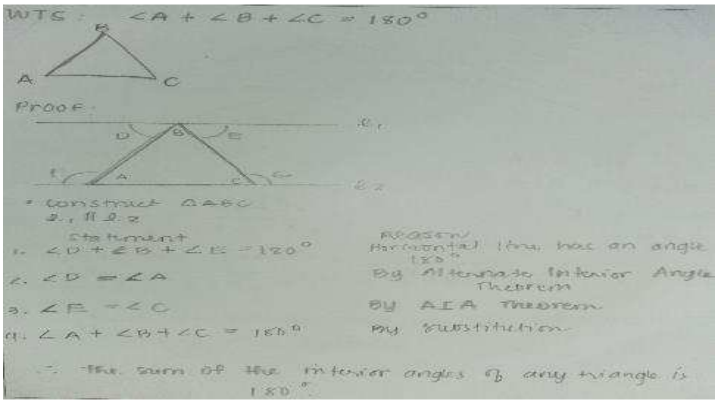


3.2.6 Axiomatic Proof Scheme

An example of axiomatic proof scheme as provided below by William.

Figure 6.

William's Proof



Figures 5 and 6 were both proof showing that the sum of the interior angles of a triangle is 180°. Both conform to the deductive reasoning proof that applies rigorous and logical reasoning. It was also found out that this mathematical statement was the easiest to prove by the respondents as shown in Table 1 (item number 1) that nine (9) responses fall under this proof scheme. It is further supported during the FGD that they chose item number 1 to be the easiest item since they previously encountered this mathematical statement. Among its responses were:

> During my practice teaching I come across with this item.
> I encountered it in college in my Geometry class.
> It was in high school that I encountered the problem (mathematical statement).
> *I'm familiar with the definition of the sum of the interior angles of a polygon that is why it's* quite easy for me.
> I taught this one during my practicum in my junior high school class.

It was also revealed in their responses during the FGD that item number 3 is the most difficult (Let n be a positive integer. Prove that it is always possible to use L-shaped triominoes to tile a board of size $2^n x 2^n$ with any one square removed.) since they encounter such mathematical statement for the first time and they forget or no idea at all the underlying mathematical concept to be used in proving the given mathematical statement. Among its responses were:

> I have no idea about the given problem (mathematical statement).
> *I'm not familiar with the given statement about the triominoes.*
> It was my first time to encounter such problem (mathematical statement*) and I don't know* where to start.
> The problem is practical and I consider it as one of *the most difficult to solve especially it's* my first time to encounter this mathematical problem.
> The item is difficult for me because it was my first time to come across with this problem on triominoes.

4. CONCLUSIONS AND RECOMMENDATIONS

In order to investigate the research questions, the respondents were engaged in constructing proofs for various mathematical statements and focused group discussion. It was documented that pre-service secondary teachers had limited knowledge on proof which is evident of no attempt in making proofs for some of the items in the proving task. This result indeed aligned with existing literature on the capabilities of pre-service teachers in constructing proofs (Likando & Ngoepe, 2014; Zeybek, 2016). Most proofs made by the pre-service teachers were empirical in nature, i.e., giving specific examples as proof for the given mathematical statement. Furthermore, only 6.43% responses (9 responses coming from a total of 140) can be classified as a valid and accepted proof using valid logical deductive reasoning. The findings of this study highlight the need for further investigation into the underlying reasons to strengthen the pre-service teachers' ability to provide quality education, in line with Sustainable Development Goals 4: Quality Education (United Nations, 2015). This is crucial, as pre-service teachers play a pivotal role in shaping the future of education and ensuring that all students have access to a quality education.

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