Application of Nonparametric Sign and Wilcoxon Sign Rank Control Charts on Animal Feed Data of PT. Japfa Comfeed Indonesia, Tbk

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Abstract

Control charts are used to see the performance of a process. Nonparametric control charts are present to overcome processes with unknown distribution patterns. This paper will show the application of nonparametric control charts using the Sign Test and Wilcoxon Sign Rank Test on the Animal Feed data of PT. Japfa Comfeed Indonesia, Tbk. The results show that the Wilcoxon Sign Rank control chart has the same effectiveness as the sign control chart.

Keywords: Sign Test; Wilcoxon Sign Test; Nonparametric; Control Chart.

1. Introduction

Statistical process control (SPC) is a tool in monitoring a process parameter. A well-known tool in SPC is the shewhart control chart. One example of a control chart is the Shewhart control chart, cumulative sum, and exponential weighted moving average. In general, these three control charts have distribution assumptions that must be met, namely normal distribution. Whereas in reality there is often a data that is not normally distributed. This nonparametric control chart itself has actually been developed for a long time including, in the book mentioned (Gibbons & Chakraborti, 2003).

This book is about inverting nonparametric statistics. This book is often used as a reference for writers who use nonparametric statistics as their test statistic. Other researchers who discuss nonparametric statistics are as follows. (Yang et al, 2011) first introduced the EWMA Sign nonparametric control which serves to detect relatively small average process shifts in data that do not meet the assumption of normality (Imam et al, 2014) developed the consistency and limits of the t-test with sign and wilcoxon sign rank test. (Harris & Hardin, 2013) developed wilcoxon signed rank and wilcoxon mann-whitney ranksum. (King & Eckersley, 2019) introduced WSR as a hypothesis test that can be applied to one-sample data or two-sample paired data. (Wilcox, 2003) discussed rank-based and nonparametric methods. (Riffenburgh, 2006) applied sign rank to the normal approximation that different distributions have zero median. (Raza et al, 2020) they wrote the EWMA statistic based on the sign nonparametric control chart to monitor the process deviation from the target. (Ali et al, 2020) the researchers designed the EWMA sign nonparametric control chart based on the sampling technique on rank and applied it in the industrial field and proposed a control chart without distribution assumptions to monitor the median of a process based on the sign test statistic.

Therefore, this paper will discuss the sign test and Wilcoxon sign rank control charts whose results will be applied to the livestock sector, namely, animal feed products at PT. Japfa Comfeed Indonesia, Tbk Makassar Unit.

2. Sign Test Control Chart

Control chart is one of the statistical quality control tools that graphically displays an overview of the behavior of a process. The function of control charts in the industrial world as a tool to monitor a production process in order to remain in statistical quality control so as to maintain the quality level of a product. The control chart is composed of control lines, namely the center line which shows the average value of quality characteristics, Upper Control Limit (UCL) and Lower Control Limit (LCL) which are used as the basis for measurement to detect out of control signals when there is a shift in the average production results (Abbasi, 2010).

Based on the data used, control charts are classified into 2 types, namely Montgomery (2009):

1. Attribute control charts are control charts for monitoring quality characteristics that are classified by product properties such as defects and non-defects.
2. Variable control chart is a control chart to monitor product quality characteristics obtained from measurements. The data required for the application of this control chart is variable data such as length, weight, volume, and others.

Making a control chart requires a mean value (μ) that can be known or unknown. The previously known μ value is a value that has been set by the company to be used as a quality measurement standard. But in practice, most of the μ values are unknown so that first an estimation is made to get the μ value. If $\bar{X}_1, \bar{X}_2, \bar{X}_3, \dots, \bar{X}_n$ is the average of each of the 1st, 2nd, ... , n samples, then the value of μ can be estimated from the average of all samples $\bar{\bar{X}}$ which is calculated by:

$$\mu = \bar{\bar{X}} = \frac{\bar{X}_1, \bar{X}_2, \bar{X}_3, \dots, \bar{X}_n}{n} \quad (1)$$

With n being the number of samples (Putri, 2011).

Shewhart sign control chart is a Shewhart control chart using a nonparametric method formed from sign test statistics to overcome data that does not meet the assumption of normality. Suppose X_j is the observation data from each sample (i) of size n and μ is the mean value of all samples. The initial procedure for making a shewhart Sign control chart is to perform a sign test statistic defined by the following equation (Amin et al, 2014):

$$SN_j = \sum_{j=i}^{n} sign(X_j - \mu_0) = \begin{cases} 1; \ if \ X_j - \mu_0 > 0 \\ 0; \ if \ X_j - \mu_0 \leq 0 \end{cases} \quad (2)$$

SN_j is binomially distributed with parameter n and p = {X_{ij} >0} = 0.5 for a controlled process. The points of the Shewhart sign control chart plot are expressed by the following equation:

$$T_i = \frac{SN_j + n}{2} \quad (3)$$

The control limits of the Shewhart sign control chart are defined as follows (Park, 2013):

$$UCL = n - b\left(\frac{\alpha}{2}, n, \frac{1}{2}\right) \quad (4)$$

$$CL = \frac{n}{2} \quad (5)$$

$$LCL = b\left(\frac{\alpha}{2}, n, \frac{1}{2}\right) \quad (6)$$

with;

$b\left(\frac{\alpha}{2}, n, \frac{1}{2}\right)$: the probability value of the binomial distribution forming the control limit
α : significance level of 0.0027
n : sample size

3. Wilcoxon Sign Test Control Chart

Suppose X_j is the data of each sample of size n and μ is the mean of a sample.
Further defined:

$$M_j = X_j - \mu \text{ and } I_j = \begin{cases} 1; \ if \ M_j > 0 \\ 0; if \ M_j \leq 0 \end{cases} \quad (7)$$

Suppose $|R_i|$= the rank of $|X_j - \mu|$, with i= 1, 2, … ,n
Therefore:

$$R^+ = \sum_{i=1}^{n} I_j R_i \quad (8)$$

Ronald (1974) wrote that the Wilcoxon sign rank test statistic has the following exact distribution:

$$Z = \frac{R^+ - \frac{n(n+1)}{4}}{\left[\frac{n(n+1)(2n+1)}{24}\right]^{1/2}} \quad (9)$$

Has a distribution alien to the normal distribution with mean 0 and variance 1.
Based on (Gibbons & Chakraborti, 2010).

$$E(R^+) = \frac{n(n+1)}{4}, and \quad (10)$$

$$Var(R^+) = \frac{n(n+1)(2n+1)}{24} \quad (11)$$

The control chart for R+are.

$$-Z_{\propto/2} < Z < Z_{\propto/2} \quad (12)$$

$$-Z_{\frac{\propto}{2}} < \frac{R^+ - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}} < Z_{\frac{\propto}{2}} \quad (13)$$

if $\propto$= 0,0027, then $Z_{\propto/2}$= 3, so

$$-3 < \frac{R^+ - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}} < 3 \quad (14)$$

$$-3\sqrt{\frac{n(n+1)(2n+1)}{24}} < R^+ - \frac{n(n+1)}{4} < 3\sqrt{\frac{n(n+1)(2n+1)}{24}} \quad (15)$$

$$\frac{n(n+1)}{4} + 3\sqrt{\frac{n(n+1)(2n+1)}{24}} < R^+ < \frac{n(n+1)}{4} - 3\sqrt{\frac{n(n+1)(2n+1)}{24}} \quad (16)$$

so,

$$UCL = \frac{n(n+1)}{4} + 3\sqrt{\frac{n(n+1)(2n+1)}{24}} \quad (17)$$

$$CL = \frac{n(n+1)}{4} \quad (18)$$

$$LCL = \frac{n(n+1)}{4} - 3\sqrt{\frac{n(n+1)(2n+1)}{24}} \quad (19)$$

4. Case Study

In the case data of animal feed products at PT Japfa Comfeed Indonesia, Tbk Makassar Unit. The inspection data for animal feed products used is variable data on the percentage of fat content in animal feed products from December 2021 to January 2022. The data consists of 30 samples of sacks of animal feed products with 10 observations of the fat content contained in each product sample, so that the total observation data used is 300 data.

The shewhart sign control chart of this animal feed data using equations (4), (5) and (6) will be obtained as follows:

UCL = 8
CL = 5
LCL = 2

As for all the resulting observations, based on Figure 1, they are within the control limits, thus this Control chart can be directly used as a control chart for further monitoring data.

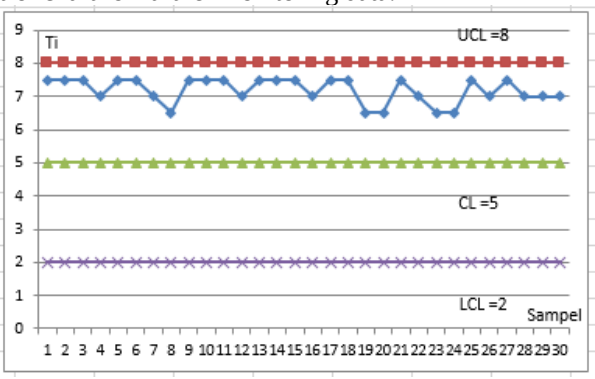

Fig. 1. Shewhart sign test control chart

The Wilcoxon Sign Rank control chart of this animal feed using equations (17), (18) and (19) will be obtained as follows:

UCL = -18.43
CL = 27.5
LCL = 56.93

As for all the resulting observations, based on Picture 2, they are within the control limits, thus this Control chart can be directly used as a control chart for further monitoring data.

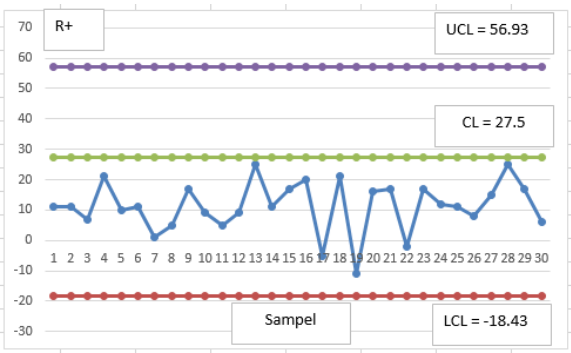

Fig. 2. Wilcoxon sign rank test control chart

Based on these two control charts, it can be concluded that the fat content of animal feed products of PT. Japfa Comfeed Indonesia, Tbk Makassar Unit is very well controlled..

5. Conclusion

The sign and Wilcoxon sign rank control charts have the same effectiveness for application to animal feed product data at PT Japfa Comfeed Indonesia, Tbk and the production process based on both control charts is concluded to be in good control process.

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References

Abbasi, S. A. (2010). On sensitivity of EWMA control chart for monitoring process dispersion. WCE 2010 - World Congress on Engineering 2010, 3, 2027–2032.

Ali, S., Abbas, Z., Nazir, H. Z., Riaz, M., Zhang, X., & Li, Y. (2020). Ondesigning non-parametric EWMA sign chart under ranked set sampling scheme with application to industrial process. Mathematics, 8(9),1-20.

Amin, R. W., Reynolds, M. R., & Bakir, S. (2014). Nonparametric quality control charts based on the sign statistic. Communications in Statistics - Theory and Methods, 24(6), 1597–1623.

Gibbons, J. D. & Chakraborti, S. (2003). Nonparametric Statistical Inference (4th ed). New York: Marcel Dekker

Gibbons, J. D. & Chakraborti, S. (2010). Nonparametric Statistical Inference (5th ed). Boca Raton: CRC Press

Harris, T., & Hardin, J. W. (2013). Exact Wilcoxon signed-rank and Wilcoxon Mann–Whitney ranksum tests. The Stata Journal (2013) 13, Number 2, pp. 337–343.

Imam, A., Usman, M., & Chiawa, M. A. (2014). On Consistency and Limitation of paired t-test, Sign and Wilcoxon Sign Rank Test. Journal of Mathematics Volume 10, Issue 1 Ver. IV, PP 01-06

King, A. P., & Eckersley, R. P. (2019). Statistics for Biomedical Engineers and Scientists. 119-145: Academic Press

Montgomery, D. C. (2009). Introduction To Statistical Quality Control. In Plastics and rubber international,10(1), 419-428.

Park, H.-I. (2013). A Note on the Median Control Chart. Communications for Statistical Applications and Methods, 20(2), 107–113.

Putri, k. i. k. (2011). Bagan Kendali Exponentially Weighted Moving Average Untuk Mean Proses. *Universitas Indonesia :* Skripsi.

Raza, M. A., Nawaz, T., Aslam, M., Bhatti, S. H., & Sherwani, R. A. K. (2020). Anew nonparametric double exponentially weighted moving average control chart. Quality and Reliability Engineering International, 36(1), 68–87.

Riffenburgh, R. H. (2006). Statistics in medicine (2th Ed). Basic of Rank, 16(1), 281-303.

Ronald, L. I. (1974). Use of a t-Statistic as an Approximation to the Exact Distribution of the Wilcoxon Signed Ranks Test Statistic. Western Michigan University

Wilcox, R. R. (2003). Applying Contemporary Statistical Techniques. PP 557-608. Los Angeles:

Yang, S. F., Lin, J. S., & Cheng, S. W. (2011). A new nonparametric EWMA Sign Control Chart. Expert Systems with Applications, 38(5), 6239–6243.